Source Depth Estimation using Matched Field Processing and Frequency-Wavenumber Transform

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***Abstract*- Matched Field Processing (MFP) is frequently used to localize underwater sources, in range and depth, using vertical arrays recording Ultra Low Frequency waves ($\leq 100Hz$). In this paper, we use matched-field techniques, guided waves propagation and signal processing tools to estimate source depth in a shallow water environment with a horizontal array. MFP is performed in the frequency-wavenumber domain: frequency-wavenumber transform (*f-k*) is the square modulus of the 2D Fourier transform in time and radial distance. This *f-k* representation provides much information on guided propagation of Ultra Low Frequency waves in shallow water: on this representation, modes can be separate. Besides, modes excitation depends on the source depth. As a result, it is possible to estimate the source depth using MFP based on modes excitation factors.**

I. INTRODUCTION

Source localization, in range and depth, is a crucial issue in underwater acoustics. Classical methods based on plane waves assumption use beamforming techniques to estimate the source bearing. These methods are unsuitable for oceanic shallow water propagation because they do not consider multipath arrivals and ocean acoustic channel complexity. Matched Field (MF) processing, which takes into account the propagation from source to receivers, has been proposed to estimate source range and depth [2]. Other methods are based on mode filtering [9] but use vertical array and harmonic source signature.

In this paper, we use matched-field techniques combined with guided waves propagation and signal processing tools to estimate source depth. The acoustic field is recorded on a horizontal array of receivers laid on the floor. Then, MF processing is performed in the frequency-wavenumber (*f-k*) domain: the *f-k* transform is the 2D Fourier transform in time and radial distance. This domain allows modes identification and separation. As guided propagation shows that modes excitation depends on source depth [5], it will be possible to estimate source depth using *f-k* transform.

After a brief discussion on MF techniques, we study modes excitation factors in a perfect wave guide to show the usefulness of the *f-k* representation. Then, *f-k* transform is used to build the cost function used in the MF localization. A study of the estimation quality is made to validate the proposed cost function. In the last part, we apply the MF estimator to estimate source depth on a real dataset.

II. SOURCE DEPTH ESTIMATION USING MATCHED FIELD PROCESSING

Matched field (MF) processing is often used to estimate ocean-bottom properties [3,8] or to localize an underwater source in range and depth using a vertical array of receivers [1,10]. MF technique consists in building a cost function depending on the set of parameters to estimate. Then the estimation of these unknown parameters is made by minimizing the cost function. One classical exemple of MF processing, used in source localization, consists in maximizing the correlation function between the acoustic field recorded by the receivers array and a predicted field due to a source at an assumed location [4]. Predicted fields are obtained by a simulation in a similar environment. A high degree of correlation between the measured field and the simulated field indicates a likely source localization. Most of the methods use the temporal signal recorded on the sensors or its spectrum to calculate the cost function [10].

Our approach is somewhat different. We use the *f-k* transform to build the cost function. Two different cost functions are studied : the first one use the whole *f-k* transform whereas the second one only takes into account the regions where modes exist on this representation. To justify these choices, let us study propagation in a perfect wave guide.

A. *Modes in the perfect homogeneous waveguide*

Let us consider a perfect wave guide (Fig. 1), e.g., a homogeneous layer of fluid between perfectly reflecting boundaries at $z=0$ and $z=D$ (reflection coefficients: -1 at the air/water interface and 1 at the water/seafloor interface). c represents the water layer velocity and ρ its density.

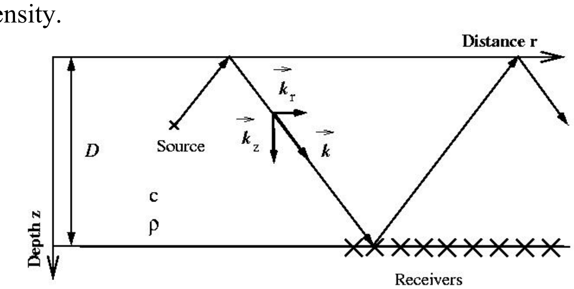

Fig. 1: Perfect wave guide

As we assume cylindrical waves, we will only study propagation in the *r-z* plane. Receivers are laid on the floor and the harmonic point source is located at depth z_S. Theorical results for a broadband source are similar. As a result, MF techniques will be performed with a broadband source in order to use all the information provided by the sensors.

The acoustic pressure $P(r,z,t)$ received at $C(r,z)$ can be expressed by $P(r,z,t) = p(r,z)\exp(-i\omega t)$ where $p(r,z)$ verify the Helmholtz equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \rho\frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial z}\right) + \frac{\omega^2}{c^2}p = -\frac{\delta(r)\delta(z-z_s)}{2\pi r} \quad (2.1)$$

with the pulsation ω. Using this expression, boundaries conditions and the technique of "separation of variables" [5, 8], we seek a solution of the unforced equation in the form $p(r,z) = \Phi(r)\Psi(z)$. Then, we express the acoustic pressure field at long range by:

$$p(r,z) = A\sum_{m=1}^{+\infty}\psi_m(z_s)\psi_m(z)\frac{\exp(ik_{rm}r)}{\sqrt{k_{rm}r}} \quad (2.2)$$

Where modes excitation factors are functions of the depth source:

$$\psi_m(z_s) = \sqrt{\frac{2}{D}}\sin(k_{zm}z_s) \quad (2.3)$$

with $k_{zm} = (2m-1)\pi/2D$. Fig. 2 represents modes excitation factors according to source depth. Two exemples at different depths : $z_{s1} = 0.2D$ and $z_{s2} = 0.5D$ are also presented on Fig. 2 and 3.

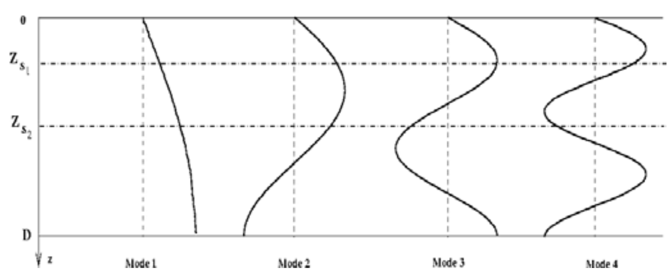

Fig. 2: Modes excitation factors

To estimate these excitation factors, we use the *f-k* transform of the seismic section recorded on the horizontal array. A previous study of this representation for guided propagation has shown that modes are separate on this representation [6]. *f-k* representation will permit to compare modes excitation factors of the model and those of the real data. As modes excitation factors depend on source depth, it will be possible to estimate the source depth (Fig. 3).

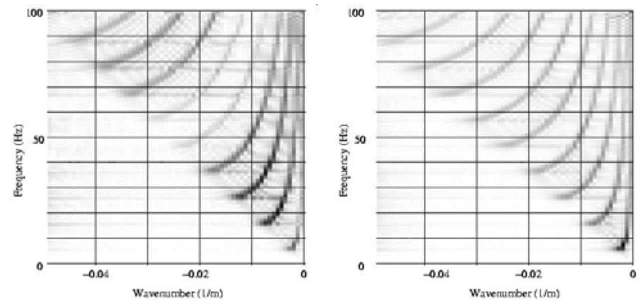

Fig. 3: *f-k* representations simulated in a perfect wave guide at two different depths : 0.2 *D* (left) and 0.5 *D* (right).

B. MFP: cost function

To compare modes excitation factors between simulated and real data, we build a cost function. Two cost functions are presented: the first one use the entire *f-k* transform whereas the second one is based on the modes existence on this representation.

1) Cost function associated to the entire f-k representation.

The *f-k* transform modulus of the real seismic section, standardized (zero mean, unit standart deviation), is considered as a $m \times n$ picture $fk_{real}(i,j)$. The cost function is the Mean Square Error (MSE) [8] between the modulus of the *f-k* representation of the real data $fk_{real}(i,j)$ and this of the simulated data $fk_{simu}(i,j)$ (also standardized):

$$MSE = \frac{1}{mn}\sum_i\sum_j \|fk_{reel}(i,j) - fk_{simu}(i,j)\|^2 \quad (2.4)$$

Using this cost function, a Signal to Noise Ratio (SNR) is built: signal is the modulus of the *f-k* representation and noise is the difference between the modulus of the *f-k* representation of the real and this of the simulated data.

$$SNR = 10\log_{10}\frac{\sum_i\sum_j \|fk_{reel}(i,j)\|^2}{MSE} \quad (2.5)$$

MF processing maximizes the SNR to estimate the source depth. The process consists in systematically placing a test point source at each depth in the guide, computing the acoustic field (replicas) at all the elements of the array and then calculating the value of the SNR between the *f-k* representation of this acoustic field and this of the real seismic section (eq. 2.5). When the test point source provides the highest value of the SNR, the source depth is estimated (Fig. 4).

Simulated fields are obtained using a finite-difference method for modeling propagation of P and SV waves in heterogeneous media. This time-distance algorithm, computed by Virieux[11], gives stable results for step velocity discontinuities, which is the case for a water layer above an elastic media. Simulations are made in an environment similar to the real environment: the environment identification is made using [7].

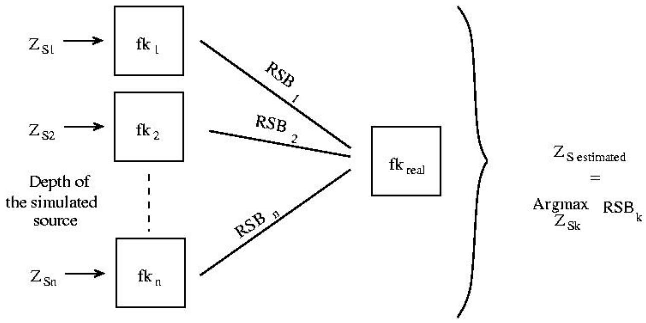

Fig. 4: Source depth estimation

1) Cost function associated to the modes on the f-k representation.

Another possibility for the cost function is to use only regions of the *f*-k representation where modes exist. As a result, the cost function will be less sensitive to noise which is present on the whole *f-k*.

A first step consists in defining a mode excitation factor based on the *f-k* transform. For each mode, a binary mask (0-1) is obtained using eq. (2.6) (where θ_1 represents the incident angle and is linked to the horizontal wavenumber by $k_{rm} = f_m / V_1 \cos\theta_1$). This mask is then dilated, this step is necessary as for real data modes will be localized on regions and not on a line.

$$\tan\Big(\frac{2\pi f_m H_1 \cos\theta_1}{V_1} - (m - \frac{1}{2})\pi\Big) = \frac{\rho_1\sqrt{\sin^2\theta_1 - (V_1/V_2)^2}}{\rho_2 \cos\theta_1} \quad (2.6)$$

Then, *f-k* representation of the section is multiplied by this dilated mask. The mean value of the *f-k* on the mask region represents the mode excitation coefficient. To compare modes excitation factors between different configurations, a normalization is made: sum of the modes excitation factors is 1. It is then possible to calculate modes excitation normalized factors $(c_1, \dots, c_n)$. The principle of coefficients building is represented on Fig 5.

As a result, the cost function is the the Mean Square Error between modes excitations factors of the real and simulated data:

$$MSE = \frac{1}{nb_{modes}} \sum_{modes} (c_{i_{simu}} - c_{i_{real}})^2 \quad (2.7)$$

As it was done with the first cost function, a Signal to Noise Ratio is built:

$$SNR = 10\log_{10}(1/MSE) \quad (2.8)$$

To maximize this SNR, we use the method presented for the first cost function: many replicas of the acoustic field (for different source depths) are simulated and compared (using modes excitation factors) to real data. Depth source is estimated using the simulation that provides the highest SNR. This cost function is less sensitive to noise (as it only takes into account regions where modes exist) and allows us to make a better estimation of the source depth. As a result, we will only study this cost function in the following.

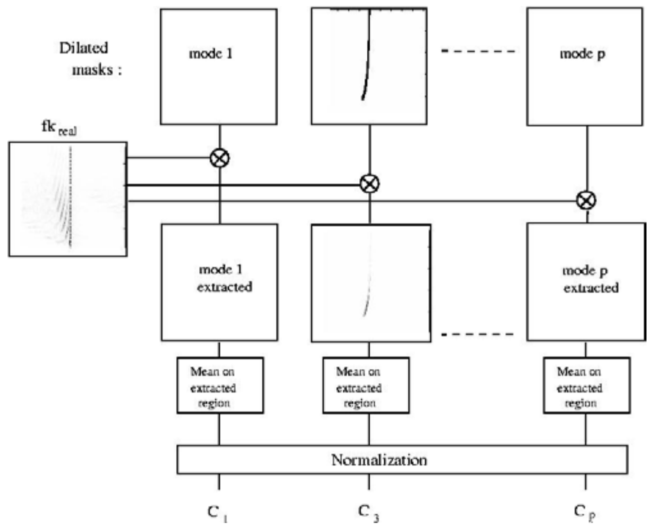

Fig. 5: Modes excitation factors using *f-k* transforms

C. Estimation quality

Theorical study of guided propagation has shown that modes excitation depends on the depth source. That leaded us to build the cost function presented in II.B.2 based on modes excitation coefficients. It is now necessary to verify that this cost function really permits an estimation of the source depth.

To do it, many simulations are made with sources at different depths. The environment is a Pekeris wave guide presented on Fig. 6.

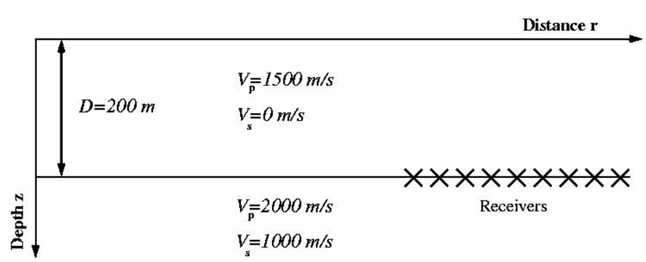

Fig. 6: Environment used to simulate a Pekeris wave guide

These simulations will show that the established cost function actually permits to compare different source depths. For each pair of simulations, the SNR is represented according to the vertical distance between the two sources. Results are presented on Fig. 6: for near sources, the SNR is high whereas for far sources, it is low. That shows that the SNR criterion is relevant to compare source depths. As a result, by comparing real data to simulated it will be possible to estimate the source depth.

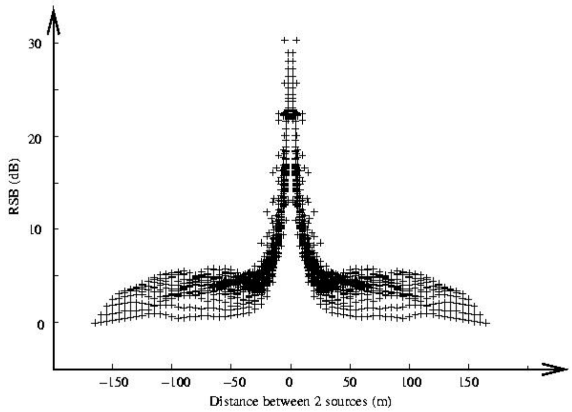

Fig. 7: SNR according to the vertical distance between sources

III. APPLICATION ON REAL DATA

Techniques described above are now used on a real dataset to estimate the source depth. The experimental geometry is shown on Fig. 8. The source is an air gun moving from one location to another and making one shot every 25 m. The receiver is a 4-components Ocean Bottom Seismometer (OBS) which provides the three components of the displacement and the pressure field. As a result, field dataset is recorded on a synthetic antenna of 240 Ocean Bottom Seismometers (OBS) laid on the North Sea floor. This geometry creates synthetic aperture and is equivalent to that presented on Fig. 9. In this application, the hydrophone is mainly used. Spatial sampling and time sampling are respectively 25 m and 4 ms.

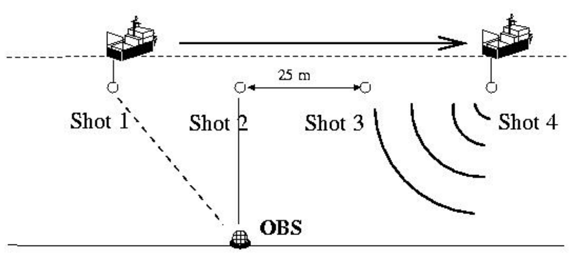

Fig. 8: Geometry of the experiment

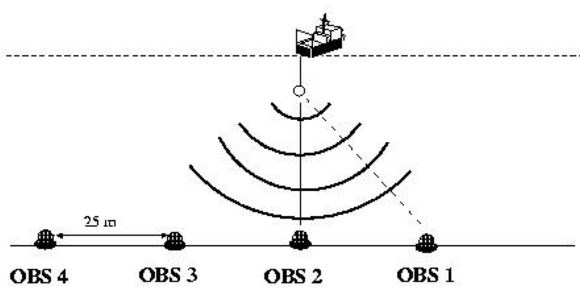

Fig. 9: Equivalent geometry: synthetic apreture created by the displacement of the source

Fig. 10 and 11 present the time-distance (after correction of the time delay due to water propagation) and frequency wavenumber plots of the real dataset. Using this representation, modes excitation factors are calculated (Fig. 12). A set of simulations is realized: Fig. 12 also shows some examples of modes excitation factors for different source depths. For each simulation, the SNR is calculated (Fig. 13). The source depth estimation is given by the depth that maximizes the SNR: we find $z_{estimated} = 17m$. We do not have the exact value of the source depth but as the source was a air gun it was bewteen 10 and 20m which is consistent with the estimated depth.

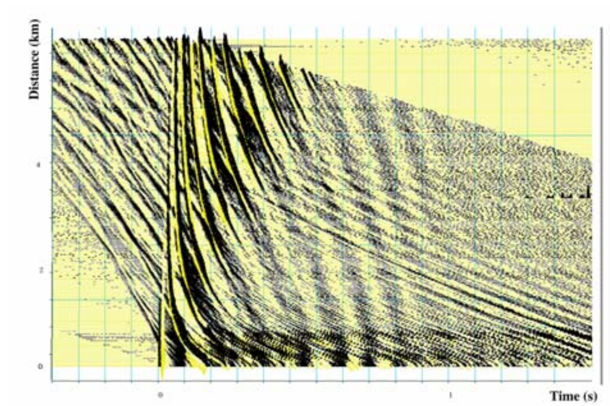

Fig. 10: time-distance representation of the seismic section

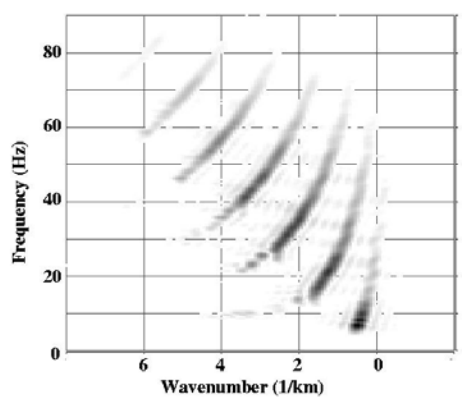

Fig. 11: *f-k* representation of the section recorded on the horizontal array

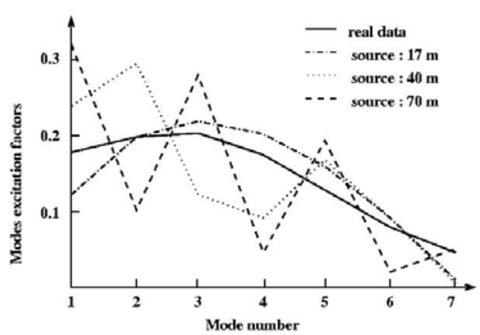

Fig. 12: Modes excitation factors (normalized) of the reeal data and of some simulated data

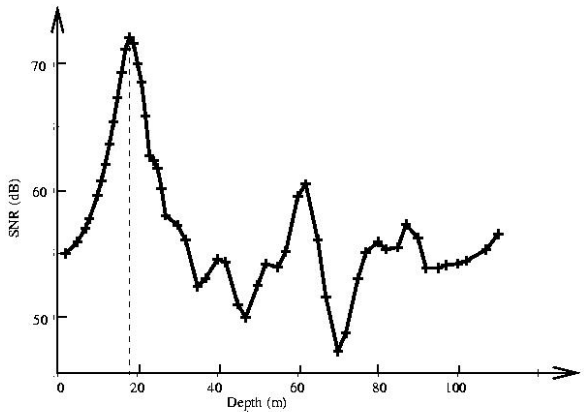

Fig. 13: SNR according to the simulated source depth

V. CONCLUSION

Study of the modal propagation shows that modes excitation factors depend on depth source. Using this property, we developp a source localization method in Ultra Low Frequency underwater acoustics. The method is perfomed in the frequency-wavenumber domain: we build a cost function based on modes excitation factors and minimize this function to estimate the source depth. This method can be used in shallow water environment (where guided propagation is preponderant). We applied it on real data and obtained satisfactory results.

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